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Entropy Balance and Performance Characterization of the Narrow Basic Pulse-Tube Refrigerator

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The basic pulse-tube refrigerator is studied using near-isothermal theory. The theory has been extended to include evaluation not only of the enthalpy flux, but also of entropy. This allows for dividing the enthalpy flux into heat and work at the extremities of the tube, and as a result, the pulse-tube performance can be determined. Except for a simple numerical integration, this study proceeds entirely in closed form, so that an investigation of the complete parameter space can be performed fairly easily. Since the device is inherently irreversible, the coefficient of performance is limited to values lower than the Carnot limit, and the results show that coefficients of performance up to about 50% of the limit are possible, but for relatively small temperature differences, outside of the cryogenic range. Still, in the usual arrangement with regenerator, a properly designed basic pulse-tube, with high-pressure amplitude and a suitable ratio between the volume of the tube and the volume of the cooler or reservoir, may conceivably remain competitive with the orifice-type device, in which, in contrast with the basic pulse-tube, an adiabatic loss at the cold end is unavoidable.

Nomenclature

c_p = specific heat at constant pressure
c_{ref} = speed of sound at T_{ref}
c_v = specific heat at constant volume
d = wall thickness
Fo = Fourier number for the wall
$\dot{H}$ = enthalpy flux
$\dot{h}$ = $\dot{H}/\int_0^1 (dp_0/dt)^2\, dt$
$I(x)$ = $\int_0^x (1/T_0)\, dx$
k = thermal conductivity
L = dimensionless length of the tube
L_{ref} = volume of the heat exchanger or reservoir, divided by the tube cross section
M = reference Mach number
$\dot{m}(x, t)$ = total mass flow rate at t and x
p = pressure
Q = total heat transfer at the warm end
R = tube radius
r = radial coordinate
$\dot{S}$ = entropy flux
s = specific entropy
T = temperature, dimensionless
$\bar{T}$ = mean temperature defined by Eq. (6)
T_{ref} = reference temperature
t = time
U = velocity at the end
u = longitudinal velocity
v = radial velocity
$W(x)$ = work done by the fluid on the left onto the fluid on the right of x
w = $W/\int_0^1 (dp_0/dt)^2\, dt$
x = longitudinal coordinate
α = heat diffusivity, $k/\rho c_p$
α_m = heat diffusivity of the wall material
γ = ratio of specific heats, c_p/c_v
δ = ratio of the thermal masses of the fluid and the wall
ε = small dimensionless number, $R^2/\alpha\tau$
μ = dynamic viscosity
ν = kinematic viscosity
ρ = density of the cycle fluid
τ = period

Subscripts
i = discretization counter characterizing x_i
L = corresponding to the left end of the tube
m = corresponding to the wall
R = corresponding to the right end of the tube
0 = leading-order term in perturbation series
11 = term of order ε in perturbation series
12 = term of order M^2/ε in perturbation series
13 = term of order δ in perturbation series
21 = term of order ε^2 in perturbation series
23 = term of order δ^2 in perturbation series

Introduction

PULSE-TUBE refrigeration has emerged as a potentially attractive approach in cryogenic refrigeration.[1] This article presents an analysis of the basic pulse-tube refrigerator, using near-isothermal theory, which is valid in the narrow tube limit. As shown before, the basic theory determines the enthalpy flux, temperatures, velocities, and pressures.[2–4] It is now extended to include an entropy balance, from which full performance characterization, including coefficient of performance and refrigeration, is obtained.

The near-isothermal model is suitable for studying basic pulse-tube refrigeration, because it includes a full representation of the relevant physics. The theory considers the effect of large pressure fluctuations, it takes into account the transverse velocity and temperature profiles, and it is based on an accurate representation of the transverse heat transfer within the fluid, so that all of the key ingredients for pulse-tube refrigeration are included.

This article is concerned only with thermal performance of the tube proper, coupled with the simplest possible model for the other components. The heat exchanger at the warm end of the tube is assumed to be ideal, no matter how small or large. A detailed study of the compressor end is not included. Even though the near-isothermal theory permits an evaluation of the drop in efficiency that results from pressure gradients caused

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by viscous losses, this has not been taken into account either. (To do justice to this topic would require a complete and separate study. But the pressure gradients merely require additional work to be provided at the cold end, which lowers efficiency without affecting the specific output. Furthermore, in the typical case, this effect is small compared with thermal losses.) The results show the crucial, and complex, role of the cooler volume, at the warm end of the tube.[5] A complete quantification of that effect is presented here.

This article contains three main parts: 1) a summary of the current theory, 2) the new theory concerned with entropy balance, and 3) a performance analysis, including the effect of the cooler volume.

Near-Isothermal Theory

Formulation and Assumptions

The near-isothermal solution to small Mach number oscillating flow in a tube, between a warm and a cold region, and with large pressure oscillations, has been proposed and discussed in detail elsewhere.[2-4] The basic idea is that, in small flow passages compared to the heat penetration depth and walls with large thermal mass, only small temperature fluctuations occur. This allows for a perturbation solution in which the leading-order flow is isothermal. In the isothermal limit, temperatures are time independent and uniform across the tube, and their longitudinal profile remains indeterminate. For arbitrary longitudinal temperature profiles, the solution determines the velocity profiles, the pressure fluctuation, the longitudinal temperature profile, the smaller, transverse, and transient temperature fluctuations, and the local longitudinal enthalpy flux. Finally, conservation of energy requires that at the periodic regime, the total enthalpy flux over one period must necessarily be uniform along the tube, resulting in a condition that determines the longitudinal temperature profile.

The basic mechanism, whereby the current theory predicts refrigeration, is fundamentally the same as that proposed for thermoacoustics by Rott[6-9] and Merkli and Thomann,[10] and applied to pulse-tubes, for instance, by Lee et al.[11] But the scaling and the region in the parameter space that the current model considers are distinctly different, and in some respects more suitable to basic pulse-tubes. One key advantage of the current theory is that it yields the equilibrium solution.

Other basic pulse-tube analysis techniques include de Boer's,[5,12] which is valid for arbitrary tube diameter, but at the cost of assuming that the processes can be separated into distinct phases of motion without heat transfer, followed by heat transfer without motion, which is not very realistic. Mirels[13,14] proposed a linearized model for gas springs and pulse-tubes, using assumptions similar to the current ones for length, arbitrary diameters, and an outside wall in contact with ambient. Storch and Radebaugh[15] have studied orifice-type pulse-tubes, neglecting all diffusive processes. None of these models (except Rott's[6]) accounts for nonuniform transverse velocities; they target systems with larger tubes, and as a result, they are not applicable for the purposes of the current study.

This section presents a brief overview of the theory, for temperature-independent μ and k, and neglecting longitudinal conduction in the wall. (For the more general case, we refer to Bauwens.[2-4])

The main assumptions are the following:

1) The length swept by the fluid motion is of magnitude comparable with the tube length. Longitudinal velocities are thus of magnitude L_{ref}/τ and L_{ref}, a yet arbitrary length scale of the same order as the tube length that will be specified when the results are applied to a specific device. Dimensionless velocities U_L and U_R at the boundaries are then of order unity. (In contrast, acoustic theory assumes a negligible sweep.)

2) The length of the tube is much larger than R. (It is shorter than the acoustic length.) The radial coordinate is scaled by R. All contributions of order R^2/L_{ref}^2 are negligible.

3) Absolute temperature differences between the two ends are of order unity. Using a yet unspecified temperature scale T_{ref}, the temperatures at both ends are of order unity.

4) The heat penetration depth in the gas is large compared to the tube diameter. Thus, $\varepsilon = (R^2/\alpha\tau) << 1$, α being the thermal diffusivity of the fluid. (Acoustic theory is valid for arbitrary penetration depth.)

5) The thermal mass of the wall is larger than the thermal mass of the fluid. Thus, $\delta = (\rho c_p/\rho_m c_m) << 1$.

6) For simplicity, the current study is restricted to the case of a wall thickness of the same order as the radius and that contributes in its entirety to the useful thermal mass, so that $Fo = \alpha_m\tau/R^2$ is at least of order unity. The study can be extended to walls, still contributing in their entirety to the useful thermal mass, but of different magnitudes[4] (see also Ref. 16).

7) Longitudinal conduction in the wall is negligible: $\tau\alpha_m/L_{\text{ref}}^2 = Fo(R^2/L_{\text{ref}}^2) << 1$.

8) Spatial pressure gradients are of a magnitude smaller than the amplitude of the temporal pressure fluctuation, which is comparable to the mean pressure, so that $\mu L_{\text{ref}}^2/\tau R^2 << P$. Introducing M, based upon the velocity scale L_{ref}/τ and the speed of sound at the reference temperature T_{ref}, then $M^2 = (L_{\text{ref}}^2/\tau^2 c_{\text{ref}}^2) << (R^2/Pr\alpha\tau) = Re_{\text{ref}}R/L_{\text{ref}} = \varepsilon/Pr$. But $R^2/\alpha\tau = \varepsilon << 1$, so that $PrM^2 << \varepsilon << 1$.

All contributions of order R^2/L_{ref}^2 are neglected entirely. Small contributions depending upon δ, ε, and M are dealt with by expanding the dependent variables in asymptotic series. (The solution remains virtually unchanged for temperature-dependent diffusion coefficients or accounting for larger longitudinal conduction in the wall.[2-4])

Periodic Boundary Value Problem

A periodic solution $u(x, r, t)$, $v(x, r, t)$, $p(x, r, t)$, $\rho(x, r, t)$, $T(x, r, t)$, and $T_m(x, r, t)$ is sought, with period 1, satisfying Eqs. (1). As shown in Fig. 1, u, v, p, ρ, and T are defined on a cylindrical domain $x \in [0, 1]$ and $r \in [0, 1]$. T_m is defined on $x \in [0, 1]$ and $r \in [1, 1 + d/R]$. Equations (1) are the conservation laws of gasdynamics, for mass, momentum, and energy, plus the energy equation for the wall, and respective boundary conditions, as shown in Fig. 1. The variables have been scaled as follows: x by L_{ref}, r by R, t by τ, u by L_{ref}/τ, v

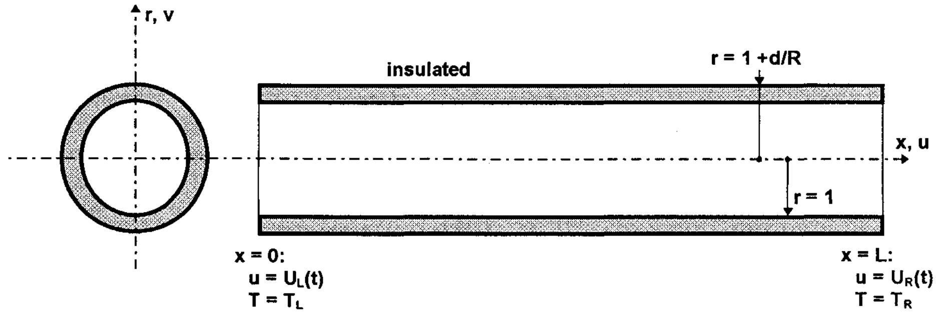

Fig. 1 Solution domain, coordinates, and boundary conditions.

by R/τ, p by P, T by T_{ref}, and density, by its value at P and T_{ref}. L_{ref} and T_{ref} remain arbitrary at this point, but specific values will be chosen later:

$$M^2\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{1}{\gamma}\frac{\partial p}{\partial x}+\frac{PrM^2}{\varepsilon}\left[\frac{1}{r}\frac{\partial\left(r\frac{\partial u}{\partial r}\right)}{\partial r}+\mathcal{O}(R^2/L^2)\right] \tag{1a}$$

$$\frac{\partial p}{\partial r}=\mathcal{O}\left(M^2\frac{R^2}{L^2}\right) \tag{1b}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{1}{r}\frac{\partial(\rho v r)}{\partial r}=0 \tag{1c}$$

$$\varepsilon\left[\frac{1}{\gamma}\frac{\partial(\rho T)}{\partial t}+\frac{1}{r}\frac{\partial(rvp)}{\partial r}+\frac{\partial(up)}{\partial x}\right]+\mathcal{O}(M^2)=\frac{1}{r}\frac{\partial\left(r\frac{\partial T}{\partial r}\right)}{\partial r}+\mathcal{O}\left(\frac{R^2}{L^2}\right) \tag{1d}$$

$$\frac{\partial T_m}{\partial t}=Fo\,\frac{1}{r}\frac{\partial\left(r\frac{\partial T_m}{\partial r}\right)}{\partial r}+\mathcal{O}\left(Fo\,\frac{R^2}{L^2}\right) \tag{1e}$$

$$p=\rho T \tag{1f}$$

$$u(0,r,t)=U_L(t) \tag{1g}$$

$$v(0,r,t)=0 \tag{1h}$$

$$T(0,r,t)=T_L \tag{1i}$$

$$T_m(0,r,t)=T_L \tag{1j}$$

$$u(L,r,t)=U_R(t) \tag{1k}$$

$$v(L,r,t)=0 \tag{1l}$$

$$T(L,r,t)=T_R \tag{1m}$$

$$T_m(L,r,t)=T_R \tag{1n}$$

$$u(x,1,t)=0 \tag{1o}$$

$$v(x,1,t)=0 \tag{1p}$$

At the interface gas-tube ($r = 1$):

$$T(x,1,t)=T_m(x,1,t) \tag{1q}$$

$$\delta\frac{\partial T}{\partial r}=\varepsilon Fo\frac{\partial T_m}{\partial r} \tag{1r}$$

$$\frac{\partial T_m}{\partial r}=0 \text{ at the outer tube wall, } r=1+\frac{d}{R} \tag{1s}$$

$$\int_0^1 p\left(\frac{L}{2},0,t\right)\mathrm{d}t=1 \tag{1t}$$

Perturbation Series

Since $PrM^2/\varepsilon << 1$, the largest independent small parameters present in Eqs. (1) are ε, δ, and PrM^2/ε. δ only appears in equations that involve T_m. Hence, the perturbation series

$$p=p_0+\varepsilon p_{11}+(PrM^2/\varepsilon)p_{12}+\varepsilon^2 p_{21}+\cdots \tag{2a}$$

$$\rho=\rho_0+\varepsilon \rho_{11}+(PrM^2/\varepsilon)\rho_{12}+\varepsilon^2 \rho_{21}+\cdots \tag{2b}$$

$$T=T_0+\varepsilon T_{11}+\delta T_{13}+\varepsilon^2 T_{21}+\cdots \tag{2c}$$

$$T_m=T_{m0}+\varepsilon T_{m11}+\delta T_{m13}+\varepsilon^2 T_{m21}+M^2T_{m22}+\delta\varepsilon T_{m23}+\cdots \tag{2d}$$

$$u=u_0+\varepsilon u_{11}+\varepsilon^2 u_{21}+\cdots \tag{2e}$$

$$v=v_0+\varepsilon v_{11}+\varepsilon^2 v_{21}+\cdots \tag{2f}$$

Leading-Order Problem

This is the Schmidt problem.[17] Replacing the dependent variables by the perturbation series, Eqs. (2), into Eqs. (1), and collecting leading-order terms, one obtains

$$\frac{\partial p_0}{\partial x}=0 \tag{3a}$$

$$\frac{\partial p_0}{\partial r}=0 \tag{3b}$$

$$\frac{\partial \rho_0}{\partial t}+\frac{\partial(\rho_0 u_0)}{\partial x}+\frac{1}{r}\frac{\partial(\rho_0 v_0 r)}{\partial r}=0 \tag{3c}$$

$$\frac{1}{r}\frac{\partial\left(r\frac{\partial T_0}{\partial r}\right)}{\partial r}=0 \tag{3d}$$

$$\frac{\partial T_{m0}}{\partial t}=Fo\,\frac{1}{r}\frac{\partial\left(r\frac{\partial T_{m0}}{\partial r}\right)}{\partial r} \tag{3e}$$

$$p_0=\rho_0 T_0 \tag{3f}$$

$$u_0(0,r,t)=U_L(t) \tag{3g}$$

$$v_0(0,r,t)=0 \tag{3h}$$

$$T_0(0,r,t)=T_{m0}(0,r,t)=T_L \tag{3i}$$

when

$$U_L(t)>0 \tag{3j}$$

$$u_0(L,r,t)=U_R(t) \tag{3k}$$

$$v_0(1,r,t)=0 \tag{3l}$$

$$T_0(L,r,t)=T_{m0}(L,r,t)=T_R \tag{3m}$$

when

$$U_R(t)<0 \tag{3n}$$

$$u_0(x,1,t)=0 \tag{3o}$$

$$v_0(x,1,t)=0 \tag{3p}$$

$$T_0(x,1,t)=T_{m0}(x,1,t) \tag{3q}$$

$$\left.\frac{\partial T_0}{\partial r}\right|_{r=1}=0 \tag{3r}$$

$$\left.\frac{\partial T_{m0}}{\partial r}\right|_{r=1}=0 \tag{3s}$$

$$\left.\frac{\partial T_{m0}}{\partial r}\right|_{r=1+(d/R)}=0 \tag{3t}$$

$$\int_0^1 p_0\,\mathrm{d}t=1 \tag{3u}$$

From Eqs. (3a) and (3b), p_0 depends upon time only. The wall temperature problem has the unique periodic solution $T_{m0} = T_{m0}(x)$. In the gas, the only solution to Eq. (3d) with T_0 finite at $r = 0$ is $T_0 = T_0(x, t)$. But since at the interface wall-gas, according to Eq. (3q), temperature is continuous, $T_{m0} = T_0 =$

$T_0(x)$. Both temperatures are equal, time independent, and uniform in r. They only vary with x. To leading order, the flow is isothermal. Replacing density by its value from the state equation, $\rho_0 = p_0(t)/T_0(x)$, in continuity, Eq. (3c)

$$\frac{\mathrm{d}p_0}{\mathrm{d}t} + T_0 p_0 \frac{\partial(u_0/T_0)}{\partial x} + p_0 \frac{1}{r}\frac{\partial(v_0 r)}{\partial r} = 0 \tag{4}$$

Dividing by T_0 and integrating over the whole tube volume, with velocities at the ends given by Eqs. (3g) and (3k), and temperatures, by Eqs. (3i) and (3m), then dividing by p_0, and integrating with respect to time, pressure is obtained in Eq. (5). T_0 is a function of x only, so that $\bar{T}$ defined by Eq. (6) is an absolute, but unknown, constant:

$$p_0(t) = \exp \int_0^t \left[\frac{\bar{T}U_L(t)}{T_L} - \frac{\bar{T}U_R(t)}{T_R}\right] \mathrm{d}t \tag{5}$$

$$\bar{T} = L \bigg/ \int_0^L \frac{1}{T_0(x)}\,\mathrm{d}x \tag{6}$$

Problem of Order PrM^2/ε

The largest contribution to pressure gradients is viscous stresses and is of order PrM^2/ε. Collecting terms of that magnitude in Eqs. (1a) and (1b) gives

$$\frac{1}{\gamma}\frac{\partial p_{12}}{\partial x} = \frac{1}{r}\frac{\partial\left(r\dfrac{\partial u_0}{\partial r}\right)}{\partial r} \tag{7a}$$

$$\frac{\partial p_{12}}{\partial r} = 0 \tag{7b}$$

From Eq. (7b), $p_{12} = p_{12}(x, t)$. Integrating Eq. (7a) twice with respect to r, noting that the left-hand side (LHS) is independent of r, and taking into account that velocity cannot be singular at $r = 0$, and that, from Eq. (1o), at $r = 1$ and $u_0 = 0$, one obtains

$$u_0 = \frac{1}{4\gamma}\frac{\partial p_{12}}{\partial x}(r^2 - 1) \tag{8}$$

Replacing u_0 by this value in Eq. (4), multiplying by r, and integrating with respect to r between 0 and 1, and taking into account that according to Eq. (1p), $v_0 = 0$ at $r = 1$ and that also, by symmetry, $v_0 = 0$ at $r = 0$, Eq. (9) is obtained:

$$\frac{1}{p_0}\frac{\mathrm{d}p_0}{\mathrm{d}t} = \frac{T_0}{8\gamma}\frac{\partial\left(\dfrac{1}{T_0}\dfrac{\partial p_{12}}{\partial x}\right)}{\partial x} \tag{9}$$

Integrating Eq. (4) times r between 0, and r and taking Eq. (9) into account, v_0 is found:

$$v_0 = \frac{1}{2p_0}\frac{\mathrm{d}p_0}{\mathrm{d}t} r(1 - r^2) \tag{10}$$

The boundary conditions, discussed in detail elsewhere[4] result in

$$\pi U_L(t) = \int_0^1 u_0(0, r, t) 2\pi r\,\mathrm{d}r \tag{11}$$

$$\pi U_R(t) = \int_0^1 u_0(1, r, t) 2\pi r\,\mathrm{d}r \tag{12}$$

Taking Eqs. (8), (9), and, respectively, Eqs. (11) and (12) into account, one finds that

$$\left.\frac{\mathrm{d}p_{12}}{\mathrm{d}x}\right|_0 = -8\gamma U_L \tag{13}$$

$$\left.\frac{\mathrm{d}p_{12}}{\mathrm{d}x}\right|_L = -8\gamma U_R \tag{14}$$

$$u_0 = 2T_0(r^2 - 1)\left(\frac{1}{p_0}\frac{\mathrm{d}p_0}{\mathrm{d}t}\int_0^x \frac{1}{T_0}\,\mathrm{d}x - \frac{U_L}{T_L}\right) \tag{15}$$

The global mass flow rate over the entire cross section is then given by Eq. (16):

$$\dot{m}_0 = \pi\left(p_0\frac{U_L}{T_L} - \frac{\mathrm{d}p_0}{\mathrm{d}t}\int_0^x \frac{1}{T_0}\,\mathrm{d}x\right) \tag{16}$$

The leading-order flowfield is now determined except for $T_0(x)$. The pressure gradient can also be determined.[4]

Problem of Order ε

Replacing T and T_m by their perturbation series in Eq. (1r), continuity of the heat flux at the interface fluid-wall ($r = 1$), implies that

$$\frac{\partial(\delta T_0 + \varepsilon\delta T_{11} + \delta^2 T_{13} + \varepsilon^2\delta T_{21}\ldots)}{\partial r} = Fo\frac{\partial(\varepsilon T_{m0} + \varepsilon^2 T_{m11} + \varepsilon\delta T_{m13} + \varepsilon^3 T_{m21} + \varepsilon^2\delta T_{m23} + \cdots)}{\partial r} \tag{17}$$

so that, at the interface

$$\left.\frac{\partial T_{m11}}{\partial r}\right|_{r=1} = 0 \tag{18a}$$

$$\left.\frac{\partial T_{11}}{\partial r}\right|_{r=1} = Fo\left.\frac{\partial T_{m13}}{\partial r}\right|_{r=1} \tag{18b}$$

but also, from Eq. (3r)

$$T_{11}(x, 1, t) = T_{m11}(x, 1, t) \tag{19a}$$

$$T_{13}(x, 1, t) = T_{m13}(x, 1, t) \tag{19b}$$

At the interface, gas temperatures of order ε match values also of order ε for the wall, while transverse gradients of order ε in the gas must be matched to gradients of order δ in the wall. Therefore, the problem of order ε for the gas is related to both problems of order ε and δ for the wall. These include Eq. (20), the perturbation of order ε to Eq. (1e), with Neumann boundary conditions given by Eq. (18a) and the perturbation to Eq. (1s), Eq. (21). The unique stationary or periodic solution is $T_{m11} = T_{m11}(x)$.

$$\frac{\partial T_{m11}}{\partial t} = Fo\frac{1}{r}\frac{\partial\left(r\dfrac{\partial T_{m11}}{\partial r}\right)}{\partial r} \tag{20}$$

$$\left.\frac{\partial T_{m11}}{\partial r}\right|_{r=1+(d/R)} = 0 \tag{21}$$

The problem of order ε for the gas can be described by Eqs. (22), obtained by collecting terms of order ε in the relevant Eqs. (1) with dependent variables replaced from Eqs. (2):

$$\frac{1}{\gamma}\frac{\partial(\rho_0 T_0)}{\partial t} + \frac{1}{r}\frac{\partial(r v_0 p_0)}{\partial r} + \frac{\partial(u_0 p_0)}{\partial x} = \frac{1}{r}\frac{\partial\left(r\frac{\partial T_{11}}{\partial r}\right)}{\partial r} \tag{22a}$$

$$\frac{\partial T_{m13}}{\partial t} = Fo\,\frac{1}{r}\frac{\partial\left(r\frac{\partial T_{m13}}{\partial r}\right)}{\partial r} \tag{22b}$$

$$T_{11}(x, 1, t) = T_{m11}(x) \tag{22c}$$

$$\left.\frac{\partial T_{11}}{\partial r}\right|_{r=1} = Fo\left.\frac{\partial T_{m13}}{\partial r}\right|_{r=1} \tag{22d}$$

$$\left.\frac{\partial T_{m13}}{\partial r}\right|_{r=1+(d/R)} = 0 \tag{22e}$$

$$T_{11}(0, r, t) = T_{m11}(0, r, t) = 0 \quad \text{when} \quad u_0(0, r, t) > 0 \tag{22f}$$

$$T_{11}(L, r, t) = T_{m11}(L, r, t) = 0 \quad \text{when} \quad u_0(L, r, t) < 0 \tag{22g}$$

u_0 and v_0 are replaced by their values from Eqs. (15) and (10) into Eq. (22a). Integrating twice yields T_{11}, given by Eq. (23):

$$T_{11} = \frac{r^4 - 4r^2 + 3}{8}\frac{dT_0}{dx}\left(\frac{dp_0}{dt}\int_0^x \frac{1}{T_0}\,dx - \frac{p_0 U_L}{T_L}\right) - (r^2 - 1)\frac{\gamma - 1}{4\gamma}\frac{dp_0}{dt} + T_{m11}(x) \tag{23}$$

Problem of Order ε^2

The highest order at which energy transfers are not inherently balanced for arbitrary $T_0(x)$ is ε^2. The corresponding energy equation for the fluid is the perturbation of order ε^2 to Eqs. (1d) and (24a). Continuity of the heat flux at the interface, Eq. (17), relates the flux of order ε^2 in the fluid to the flux of order $\varepsilon\delta$ in the wall, and is written as Eq. (24c). The energy equation of order $\varepsilon\delta$ for the wall is given by Eq. (24b):

$$\frac{1}{\gamma}\frac{\partial p_{11}}{\partial t} + \frac{1}{r}\frac{\partial(r v_0 p_{11} + r v_{11} p_0)}{\partial r} + \frac{\partial(u_0 p_{11} + u_{11} p_0)}{\partial x} = \frac{1}{r}\frac{\partial\left(r\frac{\partial T_{21}}{\partial r}\right)}{\partial r} \tag{24a}$$

$$\frac{\partial T_{m23}}{\partial t} = Fo\left[\frac{1}{r}\frac{\partial\left(r\frac{\partial T_{m23}}{\partial r}\right)}{\partial r}\right] \tag{24b}$$

$$\left.\frac{\partial T_{21}}{\partial r}\right|_{r=1} = Fo\left.\frac{\partial T_{m23}}{\partial r}\right|_{r=1} \tag{24c}$$

$$\left.\frac{\partial T_{m23}}{\partial r}\right|_{r=1+(d/R)} = 0 \tag{24d}$$

Integrating Eq. (24a) multiplied by $2\pi r\,dr$, with respect to r, between 0 and 1, and Eq. (24b) through the pipe wall, eliminating their right-hand sides (RHSs), integrating with respect to time over a full period, and taking into account that the solution is periodic, and that $T_{m0} = T_0$,

$$\int_0^1 dt \int_0^1 \left[\frac{\partial(u_0 p_{11} + u_{11} p_0)}{\partial x}\right] 2\pi r\,dr = 0 \tag{25}$$

$p_{11} = \rho_0 T_{11} + \rho_{11} T_0$ and $p_0 = \rho_0 T_0$ are replaced in the first term of Eq. (25). Noting that T_0 is independent of t and r, and that the mass flow rate of order ε is periodic, and finally integrating with respect to x, the absolute constant $\dot{H}_{11}$ is obtained:

$$\dot{H}_{11} = \frac{\gamma}{\gamma - 1}\int_0^1 dt \int_0^1 u_0 \rho_0 T_{11} 2\pi r\,dr \tag{26}$$

Replacing $u_0\rho_0$ and T_{11} by their values as functions of T_0, using $\dot{m}_0(x, t)$ given by Eq. (16), and computing the integrals with respect to r

$$\dot{H}_{11} = \frac{-11}{48\pi}\frac{\gamma}{\gamma - 1}\frac{dT_0}{dx}\int_0^1 \dot{m}_0^2\,dt + \frac{1}{6}\int_0^1 \dot{m}_0\frac{dp_0}{dt}\,dt \tag{27}$$

Replacing $\dot{m}_0$ from Eq. (16), and p_0 from Eq. (11), a first-order integro-differential equation for T_0 is obtained. Two boundary conditions are available, since temperatures are known at both ends. The problem is not overdetermined because $\dot{H}_{11}$ and $\bar{T}$ are unknown. This problem is simple to solve numerically.[2-4] For a tube closed at one end with $\dot{H}_{11} = 0$, a closed-form solution is obtained,[2-4] with the temperature going to infinity at the closed end like in the Gifford and Longsworth profile,[5,18,19] but with the exponent obtained by Müller, also in the narrow tube limit.[9,20]

If, instead of being insulated as assumed previously, the outer tube wall is in contact with a heat reservoir at a known temperature, up to order ε, the solution remains unchanged. But the RHS of Eq. (25) is no longer zero, but equals the heat transfer out of the system. The remainder of the development would remain the same except that T_0 is then known and $\dot{H}_{11}$ is no longer a constant, but it depends on x, with the local heat transfer per unit length into the system being equal to $d\dot{H}_{11}/dx$, so that the total heat transfer between x_1 and x_2 equals $\dot{H}_{11}(x_2) - \dot{H}_{11}(x_1)$.

Entropy Balance

The entropy balance that follows is a crucial new development in the theory, because it allows for the calculation of efficiency and refrigeration. Returning to the energy equation for the fluid [Eq. (1d)], subtracting the mechanical energy equation, obtained by dot-multiplying momentum by the velocity vector, from the total fluid energy equation, and taking the thermodynamic identity $T\,ds = \gamma/(\gamma - 1)\,dT - (1/\rho)\,dp$ into account, Eq. (1d) can be translated into the entropy equation, formally showing entropy creation by the diffusive mechanisms:

$$\varepsilon\rho T\left(\frac{\partial s}{\partial t} + v\frac{\partial s}{\partial r} + u\frac{\partial s}{\partial x}\right) = \frac{1}{r}\frac{\partial\left(r\frac{\partial T}{\partial r}\right)}{\partial r} + \mathcal{O}(M^2) + \mathcal{O}(R^2/L^2) \tag{28}$$

of which the first two perturbations in ε are

$$\rho_0 T_0\left(\frac{\partial s_0}{\partial t} + v_0\frac{\partial s_0}{\partial r} + u_0\frac{\partial s_0}{\partial x}\right) = \frac{1}{r}\frac{\partial\left(r\frac{\partial T_{11}}{\partial r}\right)}{\partial r} \tag{29}$$

$$(\rho_0 T_{11} + \rho_{11} T_0)\left(\frac{\partial s_0}{\partial t} + v_0\frac{\partial s_0}{\partial r} + u_0\frac{\partial s_0}{\partial x}\right) + \rho_0 T_0\left(\frac{\partial s_{11}}{\partial t} + v_0\frac{\partial s_{11}}{\partial r} + v_{11}\frac{\partial s_0}{\partial r} + u_0\frac{\partial s_{11}}{\partial x} + u_{11}\frac{\partial s_0}{\partial x}\right) = \frac{1}{r}\frac{\partial\left(r\frac{\partial T_{21}}{\partial r}\right)}{\partial r} \tag{30}$$

$[(\partial s_0/\partial t) + v_0(\partial s_0/\partial r) + u_0(\partial s_0/\partial x)]$ can be replaced in Eq. (30) by its value from Eq. (29). The continuity equation of order ε, multiplied by $T_0 s_0$, and the leading-order continuity equation multiplied by $T_0 s_{11}$ are added on the LHS. Multiplying by $2\pi r$ and integrating over the whole tube cross section, and then in time over the period, Eq. (31) is found:

$$T_0 \frac{d\dot{S}_{11}}{dx} = 2\pi \frac{\gamma}{\gamma - 1} \int_0^1 r \left.\frac{\partial T_{21}}{\partial r}\right|_{r=1} dt - 2\pi \frac{\gamma}{\gamma - 1} \int_0^1 dt \int_0^1 \frac{T_{11}}{T_0} \frac{\partial \left(r \frac{\partial T_{11}}{\partial r} \right)}{\partial r} dr \tag{31}$$

in which the total $\dot{S}_{11}$ is defined by Eq. (32):

$$\dot{S}_{11} = \frac{\gamma}{\gamma - 1} \int_0^1 dt \int_0^1 (\rho_{11} u_0 s_0 + \rho_0 u_{11} s_0 + \rho_0 u_0 s_{11}) 2\pi r \, dr \tag{32}$$

Integrating Eq. (24b) over the entire wall cross section, then with time over one period, and taking Eqs. (24c) and (24d) into account, the first integral in Eq. (32) is found to vanish. The second one can be integrated by parts:

$$T_0 \frac{d\dot{S}_{11}}{dx} = 2\pi \frac{\gamma}{\gamma - 1} \int_0^1 \left.\frac{T_{11}}{T_0} \frac{\partial T_{11}}{\partial r}\right|_{r=1} dt + \frac{1}{T_0^2} \frac{\gamma}{\gamma - 1} \int_0^1 dt \int_0^1 \left(\frac{\partial T_{11}}{\partial r} \right)^2 2\pi r \, dr \tag{33}$$

In the first integral in Eq. (33), T_{11} at the wall is time independent so that it can be factored out of the integral, which can then be shown to vanish, integrating Eq. (22b) over the entire wall cross section, then with time over one period, and taking Eqs. (22d) and (24e) into account. Finally, the following result, equivalent to the statement that entropy is created by transverse conduction within the fluid, is obtained:

$$\frac{d\dot{S}_{11}}{dx} = \frac{1}{T_0^2} \frac{\gamma}{\gamma - 1} \int_0^1 dt \int_0^1 \left(\frac{\partial T_{11}}{\partial r} \right)^2 2\pi r \, dr \tag{34}$$

T_{11} is determined by Eq. (23). Replacing it by its value in Eq. (34) and evaluating the integrals with respect to r, Eq. (35) is obtained:

$$\frac{d\dot{S}_{11}}{dx} = \frac{\gamma}{\gamma - 1} \frac{11\pi}{48} \left(\frac{dT_0}{dx} \right)^2 \times \frac{1}{T_0^2} \int_0^1 \left(\frac{dp_0}{dt} \int_0^x \frac{1}{T_0} dx - \frac{p_0 U_L}{T_L} \right)^2 dt + \frac{\pi}{3} \frac{dT_0}{dx} \frac{1}{T_0^2} \int_0^1 \frac{dp_0}{dt} \left(\frac{dp_0}{dt} \int_0^x \frac{1}{T_0} dx - \frac{p_0 U_L}{T_L} \right) dt + \pi \left(\frac{\gamma - 1}{8\gamma} \right) \frac{1}{T_0^2} \int_0^1 \left(\frac{dp_0}{dt} \right)^2 dt \tag{35}$$

which can be combined with Eq. (27), yielding

$$\frac{d\dot{S}_{11}}{dx} = \frac{d}{dx} \left(\frac{1}{T_0} \right) \left(\dot{H}_{11} + \frac{\pi}{6} \int_0^1 \frac{dp_0}{dt} \frac{p_0 U_L}{T_L} dt \right) - \frac{\pi}{6} \frac{d}{dx} \left(\frac{1}{T_0} \int_0^x \frac{1}{T_0} dx \right) \int_0^1 \left(\frac{dp_0}{dt} \right)^2 dt + \frac{7\gamma - 3}{24\gamma} \frac{\pi}{T_0^2} \int_0^1 \left(\frac{dp_0}{dt} \right)^2 dt \tag{36}$$

If the tube would end at the location x, then removal of $\dot{S}_{11}$ would require rejection of a heat flux equal to $T_0 \dot{S}_{11}$, so that, because of conservation of energy, it is only the remainder of the enthalpy flux $\dot{H}_{11} - T_0 \dot{S}_{11}$, that could be converted in work W_{11} done by the system at x. Thus, $W_{11} = \dot{H}_{11} - T_0 \dot{S}_{11}$. Likewise, for reversible processes at $x = 0$, $W_{11L} = \dot{H}_{11} - T_L \dot{S}_{11L}$. Finally, dividing these expressions, respectively, by T_0 and T_L and subtracting, it is found that $W_{11L}/T_L - W_{11}/T_0 = \dot{S}_{11} - \dot{S}_{11L} - (1/T_0 - 1/T_L)\dot{H}_{11}$. [This expression is nothing but a combination of the first and second laws of thermodynamics.[21] It can be viewed as a restriction of Eq. (3) in Ref. 21 to the problem at hand.] Finally, integrating Eq. (36) between 0 and x, a relationship between the works at the ends W_{11} and W_{11L} is obtained:

$$\frac{W_{11L}}{T_L} - \frac{W_{11}}{T_0} = \frac{1}{6} \int_0^1 \frac{dp_0}{dt} \left(\frac{\dot{m}_0}{T_0} - \frac{\dot{m}_L}{T_L} \right) dt + \pi \frac{7\gamma - 3}{24\gamma} \int_0^x \frac{1}{T_0^2} dx \int_0^1 \left(\frac{dp_0}{dt} \right)^2 dt \tag{37}$$

Basic Pulse-Tube Refrigerator

The basic pulse-tube refrigerator with regenerator and with an ideal, isothermal, heat exchanger at the warm end (on the left on the figure) is represented in Fig. 2. In this arrangement, the refrigeration equals the enthalpy flux. In the absence of the regenerator, if work is delivered at the cold end, refrigeration would then be equal to $T_R \dot{S}_{11R}$, which is then less than $\dot{H}_{11}$ (Ref. 12).

In Fig. 2, for clarity, the fluxes have been indicated with sign conventions corresponding to the actual flux directions in the refrigerator. The previous formulation, however, implicitly led to a uniform, consistent sign convention, in which fluxes are positive in the direction of positive velocities, i.e., from left to right. All of the fluxes (enthalpy, entropy, and work) are then negative in the refrigerator (and positive in an engine).

The previous theory will now be applied to the tube. The warm end is assumed to be at the left, $x = 0$. But first, the heat exchanger is considered. For an isothermal heat exchanger, the Schmidt theory[17] yields a mass flow rate equal to

$$\dot{m}_L = -\pi \frac{dp_0}{dt} \frac{L_L}{T_L} \tag{38}$$

in which L_L equals the volume of the heat exchanger divided by the tube cross section, and is made dimensionless by the tube length. [If the heat exchanger is replaced by an isentropic reservoir of finite volume, with ideal, instantaneous heat exchange at the interface with the tube, then Eq. (38) remains valid, provided that L_L is understood as the same ratio as shown previously, divided by γ.]

If the heat exchanger is just a tube with the same diameter as the pulse-tube, but with wall at fixed temperature T_L, then L_L is the heat exchanger length. In that case, a slight adaptation to the previous theory would result in net heat exchanged, equal then to the enthalpy flux at the left end of the tube:

$$Q_L = -\frac{1}{6} \frac{\pi L_L}{T_L} \int_0^1 \left(\frac{dp_0}{dt} \right)^2 dt \tag{39}$$

with the minus sign indicating that Q_L represents heat transfer from the fluid. (For different arrangements, some of which may include a reservoir, arbitrary values of Q_L are theoretically possible.) Also, $W_{11L} = 0$.

Taking Eq. (38) into account, the mass flow rates along the tube become

$$\dot{m}_0 = -\pi \frac{dp_0}{dt} \left(\frac{L_L}{T_L} + \int_0^x \frac{1}{T_0} dx \right) \tag{40}$$

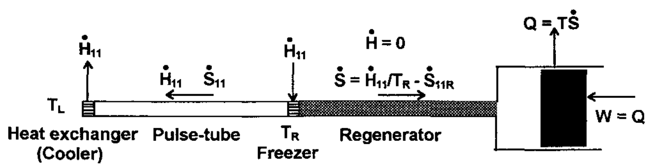

Fig. 2 Basic pulse-tube with regenerator (fluxes according to sign conventions yielding positive values).

In the RHS of both Eqs. (27) and (37), $\int_0^1 (dp_0/dt)^2\, dt$ can then be factored. As a result, it is convenient to introduce w_{11} and $\dot{h}_{11}$, defined by Eqs. (41) and (42). Also, the yet arbitrary scalars $T_{\rm ref}$ and $L_{\rm ref}$ are now set equal to the temperature at the left hand, and to the ratio volume of the heat exchanger/tube cross section, so that $L_L = 1$ and $T_L = 1$. Equations (37) and (27) then become Eqs. (43) and (44):

$$w_{11} = \frac{W_{11}}{\int_0^1 \left(\frac{dp_0}{dt}\right)^2 dt} \tag{41}$$

$$\dot{h}_{11} = \frac{\dot{H}_{11}}{\int_0^1 \left(\frac{dp_0}{dt}\right)^2 dt} \tag{42}$$

$$w_{11} = \frac{\pi}{6}\left(1 - T_0 + \int_0^x \frac{1}{T_0}\, dx - \frac{7\gamma - 3}{4\gamma} T_0 \int_0^x \frac{1}{T_0^2}\, dx\right) \tag{43}$$

$$\dot{h}_{11} = \frac{-11\pi}{48}\frac{\gamma}{\gamma - 1}\left(\int_0^x \frac{1}{T_0}\, dx + 1\right)\left[\frac{dT_0}{dx}\left(\int_0^x \frac{1}{T_0}\, dx + 1\right) + \frac{8}{11}\frac{\gamma - 1}{\gamma}\right] \tag{44}$$

By rearranging Eq. (44), an expression is obtained that can be integrated in closed form:

$$\dot{h}_{11} \frac{\frac{1}{T_0}\, dx}{\left(\int_0^x \frac{1}{T_0}\, dx + 1\right)^2} = \frac{-11\pi}{48}\frac{\gamma}{\gamma - 1} \times \left(\frac{dT_0}{T_0} + \frac{8}{11}\frac{\gamma - 1}{\gamma}\frac{\frac{1}{T_0}\, dx}{\int_0^x \frac{1}{T_0}\, dx + 1}\right) \tag{45}$$

Noting that at $x = 0$ and $T_0 = 1$, integration of Eq. (45) yields Eq. (46):

$$\dot{h}_{11}\left(\frac{1}{\int_0^x \frac{1}{T_0}\, dx + 1} - 1\right) = \frac{11\pi}{48}\frac{\gamma}{\gamma - 1}\left[\log T_0 + \frac{8}{11}\frac{\gamma - 1}{\gamma}\log\left(\int_0^x \frac{1}{T_0}\, dx + 1\right)\right] \tag{46}$$

The two relationships given by Eqs. (43) and (46) complete the results.

Solution Procedure

Instead of choosing L and T_R, which would require an iterative solution, the following procedure is implemented: a value for $\dot{h}_{11}$ is picked and a sequence of values of $I(x_i) = \int_0^{x_i} [1/T_0(x)]\, dx$ separated by small intervals ΔI is selected. For each value of the integral, $T_0(x_i)$ is calculated in closed form using Eq. (46). Then, x_i is calculated using the following second-order accurate finite difference formulation:

$$\Delta I = \frac{1}{2}\left[\frac{1}{T(x_{i-1})} + \frac{1}{T(x_i)}\right](x_i - x_{i-1}) \tag{47}$$

To compute w_{11}, the integral $\int_0^{x_i} [1/T_0(x)^2]\, dx$ is still needed. It is readily evaluated numerically, since all quantities necessary for that purpose, including temperatures, and $\Delta x = x_i - x_{i-1}$, are now available.

One advantage of this approach, besides not requiring any iteration, is that any value of x could represent L, the right end of the tube. Thus, for each value of $\dot{h}_{11}$ initially selected, one single pass through the tube length yields the whole range of temperature ratios. The entire two-dimensional parameter space can thus be investigated by scanning a range of values of $\dot{h}_{11}$. The solution is fast enough to allow for a very fine discretization, hence, an insignificant truncation error.

Results

The results include a full mapping of the performance of the basic pulse-tube, using helium ($\gamma = 1.665$), vs the two key parameters, which are the temperature ratio and the ratio tube volume/reservoir or heat exchanger volume. Figures 3 and 4 show the ratio coefficient of performance/Carnot coefficient of performance, vs the ratio tube volume/heat exchanger (or reservoir) volume, and vs the temperature ratio, for a basic pulse-tube without regenerator, corresponding to the device shown in Fig. 2, but in which the compressor is placed at the cold end, immediately beside the freezer (this may not be a practical arrangement, but it is useful for discussion and analysis). Figure 5 shows the same results as Figs. 3 and 4, but for the configuration with regenerator shown in Fig. 2. Finally, Figs. 6 and 7 show contour plots of enthalpy flux and power, also vs length ratio and temperature ratio.

The results in Figs. 3 and 4 show that while there is a region in which the basic pulse-tube without regenerator reaches a coefficient of performance (COP) higher than 50% of the Carnot value, reasonable COPs are obtainable only for relatively low temperature ratios, outside of the cryogenic range. This is

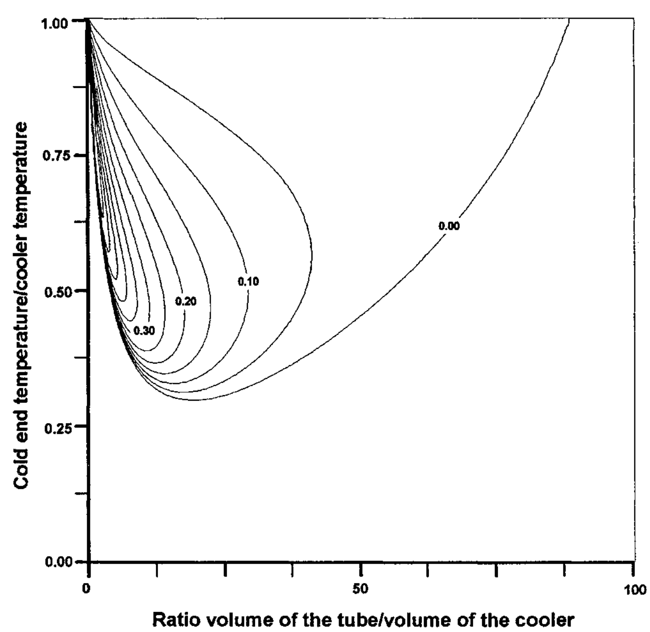

Fig. 3 Relative coefficient of performance vs ratio tube volume/ cooler volume and temperature ratio (cooler/freezer); configuration without regenerator.

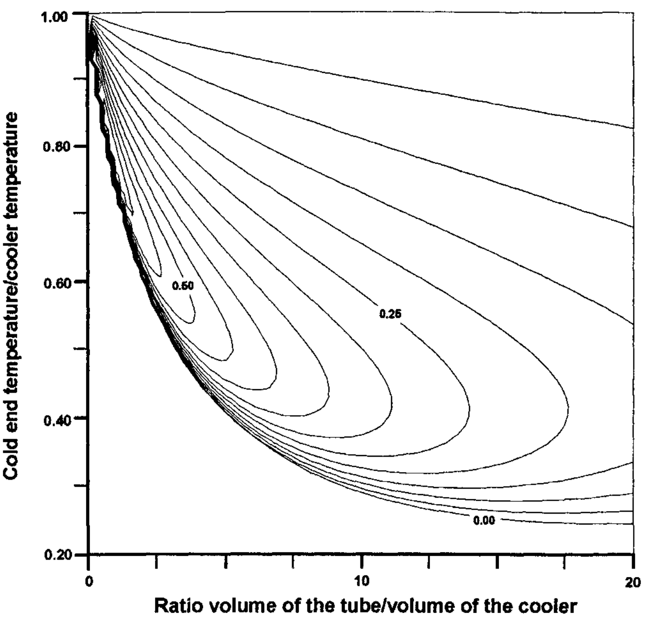

Fig. 4 Relative coefficient of performance vs ratio tube volume/cooler volume and temperature ratio (cooler/freezer); configuration without regenerator, low-lift region.

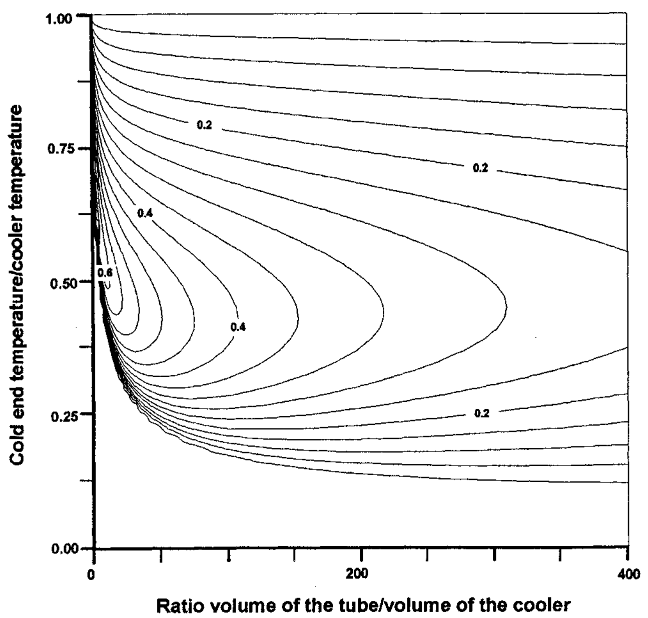

Fig. 5 Relative coefficient of performance vs ratio tube volume/cooler volume and temperature ratio (cooler/freezer); configuration with regenerator.

significant, because it is only in the range where the basic pulse-tube effect (without regenerator) contributes a positive net refrigeration that the configuration shown in Fig. 2, with the regenerator, will beat an ideal orifice-type device.[5,12] And indeed, Fig. 4 shows a better performance than the orifice-type device in the relatively low lift region (on the top left region on the plot), but a distinctly poorer performance for small values of $T_{\text{freezer}}/T_{\text{cooler}}$, at the bottom range of the chart, under which conditions the COP of the ideal orifice pulse-tube approaches the Carnot limit.

This comparison is somewhat unfair, however, because the present analysis accounts for heat transfer losses in the tube, while similar losses have been ignored in the ideal orifice device,[14] where they also occur to some extent. But more importantly, in contrast with the orifice pulse-tube, in the basic pulse-tube with regenerator, adiabatic losses at the cold end can be avoided. These losses are significant; they could easily reduce the coefficient of performance of the orifice pulse-tube by 20–30%.[22–24] To minimize the effect of the adiabatic loss, pressure amplitudes will have to be limited to a small fraction of the mean pressure in the orifice-type refrigerator, while the basic pulse-tube can be designed for larger pressure amplitudes without suffering from adiabatic losses at the cold side.

As for the heat lifted and power required, results are shown in Figs. 6 and 7. They indicate that to produce a sizable lift, a design at the edge of the validity range of the basic assump-

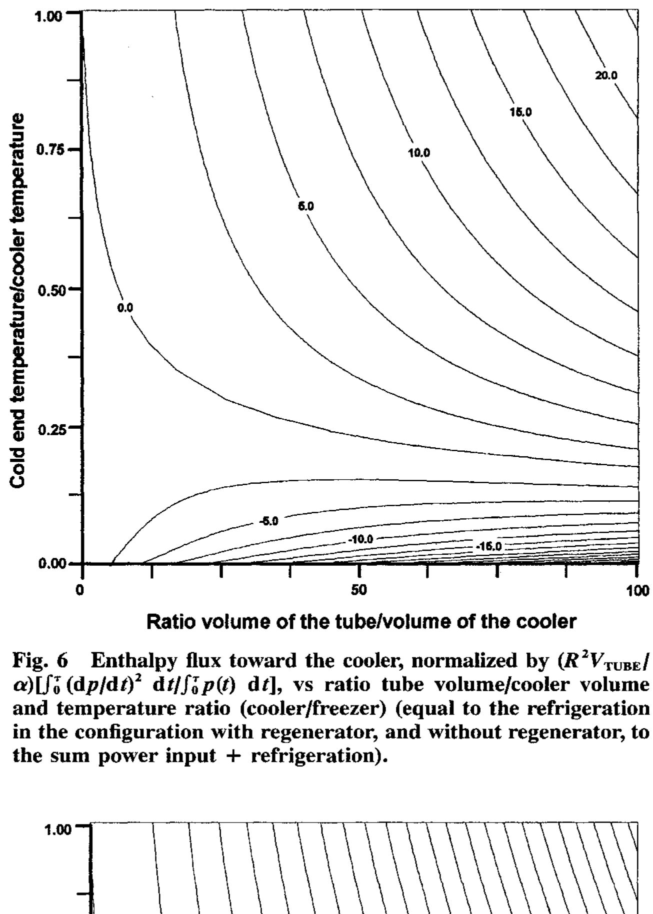

Fig. 6 Enthalpy flux toward the cooler, normalized by $(R^2 V_{\text{TUBE}}/\alpha)[\int_0^\tau (dp/dt)^2\, dt/\int_0^\tau p(t)\, dt]$, vs ratio tube volume/cooler volume and temperature ratio (cooler/freezer) (equal to the refrigeration in the configuration with regenerator, and without regenerator, to the sum power input + refrigeration).

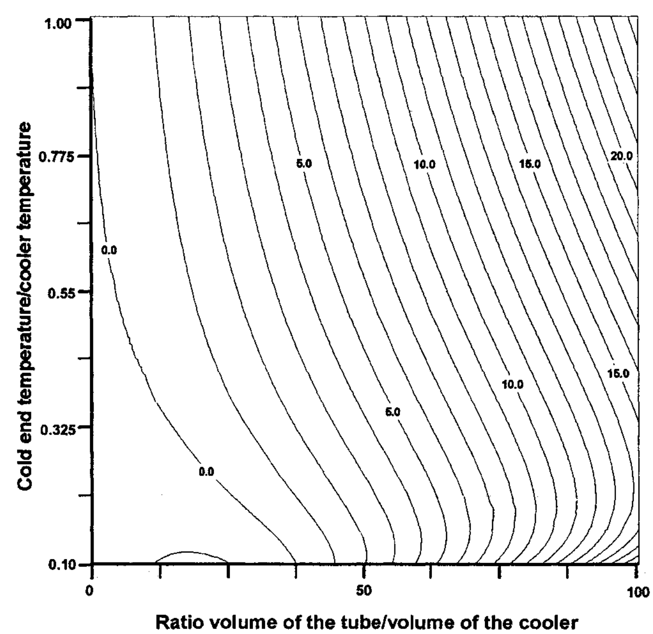

Fig. 7 Power input at the cold end, normalized by $(R^2 V_{\text{TUBE}}/\alpha)[\int_0^\tau (dp/dt)^2\, dt/\int_0^\tau p(t)\, dt]$, vs ratio tube volume/cooler volume and temperature ratio (cooler/freezer) (equals the power input without regenerator, and with regenerator, power input equals this number times $T_{\text{COMPRESSOR}}/T_R$).

tion, that the tube is smaller than the heat penetration depth, may be necessary.

The boundary between positive and negative enthalpy flux, in Fig. 6, does not match the boundary between positive and negative power, in Fig. 7. (The same sign convention as in Fig. 2 has been used in Figs. 6 and 7, with fluxes in the direction that occurs in refrigerators, and power input, being accounted for as positive.) Three distinct regions can be identified. The region where both enthalpy flux and power are positive is the region where a device without regenerator yields a positive refrigeration, as also shown in Fig. 3. In the region where the enthalpy flux is negative, but power is still positive, the tube is an energy dissipator; the basic pulse-tube with regenerator still has a positive COP, but lower than in an equivalent orifice-type device. Finally, in the region where both enthalpy flux and power are negative, the device operates as a heat engine.

The performance charts shown in Figs. 3–7 show the significance of the key design parameter in the basic pulse-tube,[5] which is the ratio tube volume/heat exchanger or reservoir volume. It is interesting to note that the lines of constant COP form closed loops starting at the point with zero length and no temperature difference, but that as the temperature difference increases, maintaining the same efficiency requires a smaller heat exchanger or reservoir volume at the warm end. It also appears that the highest COP is reached at very low lift, raising the possibility that there might be a niche for the basic pulse-tube without regenerator in low-lift refrigeration, especially if the simplicity of the device is taken into account. However, the current analysis does not include the effect of the adiabatic loss at the cold end, which occurs in this configuration because the piston is at the cold end, so that there is an adiabatic cylinder space at the cold end, and the adiabatic loss will become significant at low lift.

Overall, these results are consistent with the bulk of experimental evidence, which shows that it is fairly easy to obtain some initial pulse-tube refrigeration while the temperature difference is small, but that basic pulse-tube refrigeration does not appear to reach the higher temperature differences characteristic of cryogenics.[1]

Conclusions

The near-isothermal model proves to be a very effective tool for basic pulse-tube performance characterization. Even though it includes a fairly complex set of physics, it allows for evaluation of specific performance and efficiency over the entire parameter space, in a study that is almost entirely in closed form, except for a simple numerical integration at the end.

A full performance mapping of the basic pulse-tube refrigerator was presented. Results show perhaps surprisingly high figures for the coefficient of performance, especially since in experiments, the basic pulse-tube has never been shown to be a very effective device. But the high efficiencies only occur in a narrow range of temperature ratios that may indicate a potential application in low-lift refrigeration, but does not go down to the cryogenic range. Specific output also appears to be low, but this is largely because this study considered the narrow tube limit. (The situation is similar as with linearized acoustic models that also yield vanishing results, obtained in the limit of zero amplitude, but are routinely used in conditions under which, technically, the assumption of linearity is no longer valid.) A definitive conclusion regarding specific output will depend upon how well the results can be extrapolated outside the formal range of validity of the near-isothermal assumption. This will require experimentation.

The results show the key role played by the ratio volume of tube/volume of the heat exchanger or reservoir, and they provide a guide for sizing of the reservoir.

Finally, the results indicate that the basic pulse-tube with regenerator yields a substantially lower performance than the ideal orifice device. But, in contrast with the orifice-type pulse-tube, in the basic pulse-tube, the adiabatic loss can be avoided at the cold end. Thus, a definitive conclusion regarding the respective merits of the two arrangements will require a careful comparative study, accounting more accurately for all the significant differences.

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